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ANALYSIS OF THE LATTICE THERMAL RESISTIVITY DUE TO THE PRESENCE OF ELECTRONS AT LOW TEMPERATURES: APPLICATION TO PHOSPHORUS-DOPED Ge

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The temperature-dependence of the extra lattice thermal resistivity of a doped sample due to the presence of electrons has been studied at low temperatures for the first time by analysing the extra lattice thermal resistivity due to electrons of five samples of phosphorus-doped Ge having different carrier concentrations in the range $1.2\times10^{23} - 1.1\times10^{24}$ m^{-3} in the temperature range 1–5 K. The variation of the extra lattice thermal resistivity of a doped sample due to electrons with the parameters η^* (the reduced Fermi energy), m^* (the density of states effective mass), E_D (the deformation potential constant) and n (the carrier concentration) which are responsible for the electron-phonon scattering relaxation rate has also been analysed for the first time in the present study. A distinction has been made between non-peripheral and peripheral phonons in the present analysis. An analytical expression is reported for calculation of an approximate value of the extra lattice thermal resistivity of a doped sample due to the presence of electrons at low temperatures.

In a doped semiconductor, the presence of electrons means an extra scatterer to phonons, and the scattering of phonons may be due either to the conduction-state electrons [1] or to the bound-state electrons [24], which depends mainly on the position of the Fermi energy and the concentration of electrons. For low concentrations, the impure atoms may be regarded as independent scatterers of phonons, and phonons are scattered mainly due to virtual transitions of electrons between the singlet state and the first excited triplet state, and the lattice thermal resistivity of the doped sample is mainly due to bound-state electrons. For the higher concentrations, the impurity levels overlap with the conduction band and only a few free electrons are available. As a result, the lattice thermal resistivity of a doped sample having a higher carrier concentration is mainly due to the conduction-state electrons. Several workers [5–7] have studied the lattice thermal conductivity of doped semiconductors and it is well established that, for a doped sample having a donor electron concentration larger than 10^{23} m^{-3}, the donor levels merge with the conduction band and the scattering of phonons by the conduction-state electrons is the most relevant scattering mechanism, while the expression reported by Ziman [1] for the electron-phonon scattering relaxation rate gives a very good response to the experimental data [8] of a doped semiconductor having a carrier concentration larger than 10^{23} m^{-3}, at low temperatures.

During the study of the scattering of phonons by the conduction-state electrons, however, it has been found [1, 9, 10] that the conservation of momentum and energy requires that an electron in state $\bar{K}$ can only interact with those phonons for which the wave vector $|\bar{q}| \leq 2|\bar{K}|$. For metals, $\bar{K} = \bar{K}_F$, where $\bar{K}_F$ is the electron wave vector corresponding to the Fermi surface. In view of the above fact and following the earlier workers [9–12], the entire phonons can be divided into two groups. Group I phonons are those which have wave vector $|\bar{q}| \leq 2|\bar{K}_F|$ and can interact with electrons. Such phonons are known as non-peripheral [9–12] phonons. Group II phonons are those which have wave vector $|\bar{q}| > 2|\bar{K}_F|$ and can not scatter with electrons. These phonons are referred to as peripheral [9–12] phonons. Thus, it is clear that the extra lattice thermal resistivity of a doped sample having a carrier concentration larger than 10^{13} m^{-3} is entirely due to non-peripheral phonons. The extra lattice thermal resistivity W_e of a doped sample due to the presence of electrons can be studied by subtracting the lattice thermal resistivity of the undoped sample from that of the corresponding doped sample, i.e. $W_e = W_{\text{doped}} - W_{\text{undoped}}$, where W_{doped} is the lattice thermal resistivity of a doped sample and W_{undoped} is that before the doping.

The aim of the present work is to study the extra lattice thermal resistivity W_e of a doped semiconductor due to the presence of electrons at low temperatures for the first time by calculating the extra lattice thermal resistivity W_e due to electrons of five samples of phosphorus-doped Ge having different carrier concentrations in the range $1.2 \times 10^{23} - 1.1 \times 10^{24}$ m^{-3} in the entire temperature range 1–5 K. The variation of W_e with the parameters η^* (the reduced Fermi energy), m^* (the density of states effective mass), E_D (the deformation potential constant) and n (the carrier concentration), which are the factors responsible for the electron-phonon scattering relaxation rate, has also been studied to examine the effects of these parameters on the extra lattice thermal resistivity due to electrons. The percentage contribution $\%\ W_e$ towards the total lattice thermal resistivity has been determined to analyse the relative contribution of the extra lattice thermal resistivity of a doped sample due to the presence of electrons. An analytical expression is reported for calculation of an approximate value of W_e at low temperatures.

Theory

In spite of the fact that some refinements [13–17] have been proposed, the success of the Callaway [18] integral in explaining the experimental data of the lattice thermal conductivity is very good at low temperatures. Following Callaway, the total lattice thermal resistivity of the undoped sample at low temperature can be expressed as

$$W^{-1}_{\text{undoped}} = C \int_0^{\theta/T} \tau_p F(x)\,dx + \Delta K \tag{1}$$

where $\tau_p^{-1} = \tau_B^{-1} + \tau_{pt}^{-1} + \tau_{ph}^{-1}$, $F(x) = x^4 e^x (e^x - 1)^{-2}$,

$C = (K_B/2\pi^2V)\,(K_B/\hbar)^3$, τ_p^{-1} is the combined scattering relaxation rate for the undoped sample, τ_B^{-1}, τ_{pt}^{-1} and τ_{ph}^{-1} are the boundary [19], point-defect [20] and phonon-phonon [21] scattering relaxation rates, respectively, K_B is the Boltzmann constant, $\hbar$ is the Planck constant divided by 2π, V is the average phonon velocity, $x = \hbar\omega/(K_BT)$ is a dimensionless parameter, ω is the phonon frequency, θ is the Debye temperature of the sample under study, and ΔK is the correction term [18] due to the three-phonon normal processes, which can be neglected due to its very small contribution [22–26] compared to the contribution due to the first part in Eq. (1). Since our study is confined to low temperatures only, the expression for the above scattering relaxation rates used in the present analysis can be expressed as $\tau_B^{-1} = V/L$, $\tau_{pt}^{-1} = A\omega^4$ and $\tau_{ph}^{-1} = B\omega^2T^3$, where L is the Casimir [19] length of the crystal, and A and B are the scattering strengths due to the respective processes. It should be noted that τ_{ph}^{-1} has been ignored in the actual calculation, due to its very small values compared to the other scattering relaxation rates at low temperatures.

In a doped sample, the presence of electrons cause two types of carrier concentration-dependent scatterings besides the scatterings stated above for an undoped sample, and the electron-phonon scattering mechanism is the most important scattering process in such a sample. At low temperatures, the lattice thermal resistivity of a doped sample is mainly due to electron-phonon scattering. The scattering of phonons may be due to the bound-state [2–4] electrons or the conduction-state [1] electrons. From earlier studies [5–7], it is well established that for a doped sample having a carrier concentration less than $10^{23}\,m^{-3}$, the scattering of phonons by the bound-state electrons is more effective and the lattice thermal resistivity of such a sample is mainly due to the scattering of phonons by the bound-state electrons. On the other hand, in a sample having a carrier concentration larger than $10^{23}\,m^{-3}$, the scattering of phonons by the conduction-state electrons is the most important scattering mechanism and the lattice thermal resistivity of such a sample is mainly due to the presence of the conduction-state electrons. According to Ziman [1], the electron (conduction state) – phonon scattering relaxation rate can be expressed as

$$\tau_{ep}^{-1} = DT \ln\left\{\frac{[1 + \exp(\eta^* - (N/T) - PTx^2 + x/2)]}{[1 + \exp(\eta^* - (N/T) - PTx^2 - x/2)]}\right\} \tag{2}$$

where $D = \dfrac{E_D^2 m^{*2} K_B \delta^2}{4\pi\hbar^4 M V_L}$, $\eta^* = \dfrac{E_F}{K_BT}$, $N = \dfrac{m^* V_L^2}{2K_B}$, $P = \dfrac{K_B}{8m^* V_L^2}$

E_D is the deformation potential constant, m^* is the density of the states effective mass, η^* is the reduced Fermi energy, δ^3 is the atomic volume V_L is the longitudinal phonon velocity, E_F is the Fermi energy level, M is the mean atomic weight, and other terms have the same meaning as defined earlier.

Due to the requirement of the conservation of momentum and energy, all of the phonons can not interact with the conduction-state electrons. The phonons having

the wave vector $|\bar{q}| \leq 2|\bar{K}_F|$ can interact with the electrons and are referred to as non-peripheral [9–12] phonons, while the phonons having the wave vector $|\bar{q}| > 2|\bar{K}_F|$ can not interact with the electrons and such phonons are referred to as peripheral phonons [9–12], where $\bar{K}_F$ is the electron wave vector corresponding to the Fermi level. Thus, it is clear that the peripheral phonons can not take part in the electron-phonon scattering mechanism and it is necessary to make a distinction between non-peripheral and peripheral phonons in the calculation of the lattice thermal resistivity of a doped sample having a carrier concentration larger than 10^{23} m^{-3}. Introducing the idea of peripheral and non-peripheral phonons, following the earlier work of Dubey and Verma [10], and using Eq. (1), the total lattice thermal resistivity of a doped sample at low temperatures can be expressed as

$$W_{\text{doped}}^{-1} = C\left[\int_0^{\theta^*/T} (\tau_P^{-1} + \tau_{ep}^{-1})^{-1} F(x)\,dx + \int_{\theta^*/T}^{\theta/T} \tau_P F(x)\,dx\right] \tag{3}$$

where $\theta = \dfrac{2F\hbar V_L}{K_B}(\pi^2 n)^{1/3}$ is the characteristic temperature [10] of the sample which differentiates the peripheral phonons from non-peripheral phonons and depends mainly on the carrier concentration n, F is a constant [10] and depends on the carrier concentration n, and other terms have the same meanings as defined earlier. It should be noted that the first integral in Eq. (3) corresponds to the contribution due to non-peripheral phonons, while the contribution of the second integral is due to peripheral phonons. It should also be noted that the scattering relaxation rates due to other scattering processes are assumed to be the same as for the corresponding undoped sample, due to the fact that doping produces a negligibly small effect on these scattering relaxation rates.

The extra lattice thermal resistivity W_e of a sample due to the presence of electrons can be expressed in terms of the total lattice thermal resistivities of undoped and of the corresponding doped samples as

$$W_e = W_{\text{doped}} - W_{\text{undoped}} \tag{4}$$

Using Eqs (1) and (3) for W_{undoped} and W_{doped}, respectively, W_e can be expressed as

$$W_e = \frac{R_1 - R_2}{C R_3(R_3 - R_1 + R_2)} \tag{5}$$

where

$$R_1 = \int_0^{\theta^*/T} (\tau_B^{-1} + \tau_{pt}^{-1} + \tau_{ph}^{-1})^{-1} F(x)\,dx \tag{6}$$

$$R_2 = \int_0^{\theta^*/T} (\tau_B^{-1} + \tau_{pt}^{-1} + \tau_{ph}^{-1} + \tau_{ep}^{-1})^{-1} F(x)\,dx \tag{7}$$

$$R_3 = \int_0^{\theta/T} (\tau_B^{-1} + \tau_{pt}^{-1} + \tau_{ph}^{-1})^{-1} F(x)\,dx \tag{8}$$

Using Eq. (5) and going through a little mathematical manipulation, the extra lattice thermal resistivity W_e of a sample due to the presence of electrons can be expressed either in terms of W_{undoped}, or in terms of W_{doped}, or in terms of both, as

$$W_e = W^2_{\text{undoped}} \frac{CR}{(1 - CRW_{\text{undoped}})} \tag{9}$$

$$W_e = W^2_{\text{doped}} \frac{CR}{(1 + CRW_{\text{doped}})} \tag{10}$$

and

$$W_e = W_{\text{undoped}} W_{\text{doped}} CR \tag{11}$$

where

$$R = \int_0^{\theta^*/T} \tau_{ep}^{-1} \tau_P (\tau_{ep}^{-1} + \tau_P^{-1})^{-1} F(x) \mathrm{d}x \tag{12}$$

Thus, from a knowledge of the experimental values of W_{doped} or W_{undoped}, W_e can be estimated via Eqs (9)–(12).

Analytical expression

From the previous section, it is clear that to calculate the extra lattice thermal resistivity W_e of a doped sample due to the presence of electrons, one has to go through the numerical integration of the complicated integrals at each temperature, which is not an easy task. Therefore, there is a need to have an analytical expression to evaluate an approximate value of W_e. During the numerical analysis of the earlier-given integrals, it is found that at low temperatures either $\tau_B^{-1} > \tau_{ep}^{-1}$ or $\tau_B^{-1} < \tau_{ep}^{-1}$, while other scattering relaxation rates have smaller values compared to these two scattering relaxation rates. Therefore, the analytical expressions have been obtained under these two approximations.

Following the earlier work of the author [27–29], considering the domination of τ_B^{-1} over τ_{pt}^{-1} and τ_{ph}^{-1} due to the low temperature, and using Eq. (1), the lattice thermal resistivity W_{undoped} can be approximated as

$$W^{-1}_{\text{undoped}} \simeq C\tau_B \int_0^{\theta/T} (1 - A_2 \tau_B x^4 - B_2 \tau_B x^2) x^4 e^x (e^x - 1)^{-2} \mathrm{d}x \tag{13}$$

where

$$A_2 = A(K_B/)\hbar^4 T^4 = A_1 T^4$$

$$B_2 = B(K_B/\hbar)^2 T^5 = B_1 T^5$$

which can be expressed as

$$W^{-1}_{\text{undoped}} = C\tau_B I_4 [1 - A_2 \tau_B F_4^8 - B_2 \tau_B F_4^6] \tag{14}$$

where

$$F_n^m = I_m/I_n,\ I_r = \int_0^{\theta/T} x^r e^x (e^x - 1)^{-2} \mathrm{d}x,\ r = 1, 2, \ldots$$

Considering θ/T as ∞ due to the large value of θ and the very low values of T, the integral I_r can be evaluated with the help of the Riemann zeta function. Thus, one gets an expression for W_{undoped} as

$$W_{\text{undoped}} = \frac{15}{4\pi^4} \frac{\tau_B^{-1}}{C} \left(1 + 16\pi^4 A_2 \tau_B + \frac{20\pi^2}{7} B_2 \tau_B\right) \tag{15}$$

As stated earlier, in a doped sample at low temperatures, either $\tau_B^{-1} > \tau_{ep}^{-1}$ or $\tau_B^{-1} < \tau_{ep}^{-1}$. Thus, following Eq. (15) an expression for the total lattice thermal resistivity W_{doped} can also be obtained similarly as for W_{undoped}.

(A) If $\tau_B^{-1} > \tau_{ep}^{-1}$.

At low temperatures, Eq. (2) can be approximated as $\tau_{ep}^{-1} = DTx$. Considering $\tau_B^{-1} > \tau_{ep}^{-1}$, following Eq. (13) and using Eq. (3), the total lattice thermal resistivity W_{doped} can be approximated as

$$W_{\text{doped}} \simeq \frac{\tau_B^{-1}}{CI_4} (1 + A_2 \tau_B F_4^8 + B_2 \tau_B F_4^6 + \mathrm{d}\, \tau_B J_5/I_4) \tag{16}$$

$$\text{where } DT = d, \quad \text{and} \quad J_N = \int_0^{\theta^*/T} x^N e^x (e^x - 1)^{-2} \mathrm{d}x;$$

$N = 1, 2, 3 \ldots$ and other terms have the same meanings as defined earlier. At low temperatures, $x = (\hbar\omega/K_B T) << 1$ and $(e^x - 1)$ can be approximated as e^x, and hence, the integral I_N can be approximated as

$$J_N = \int_0^{\theta^*/T} x^N e^x (e^x - 1)^{-2} \mathrm{d}x \simeq \int_{1/T}^{\theta^*/T} x^N e^x (e^x - 1)^{-2} \mathrm{d}x = \int_{1/T}^{\theta^*/T} x^N e^{-x} \mathrm{d}x$$

$$= \mathrm{N}! \left\{ e^{-1/T} \sum_{n=0}^{N} \frac{1}{\mathrm{n}!} T^{-n} - e^{-\theta^*/T} \sum_{n=0}^{N} \frac{1}{\mathrm{n}!} \left(\frac{\theta}{T}\right)^n \right\} \tag{17}$$

Using Eqs (15), (16) and (4), the expression for the extra lattice thermal resistivity W_e of a doped sample due to the presence of electrons at low temperatures for $\tau_B^{-1} > \tau_{ep}^{-1}$ can be approximated as

$$W_e = 9.31 \times 10^{-3} \quad VK_B^{-4} \hbar^3 D T^{-2}. \tag{18}$$

(B) If $\tau_B^{-1} < \tau_{ep}^{-1}$.

For $\tau_{ep}^{-1} > \tau_B^{-1}$, Eq. (3) can be approximated as

$$W_{\text{doped}}^{-1} = \frac{C}{d} J_3[1 - (A_2/d)H_3^6 - (B_2/d)H_3^4 - (\tau_B^{-1}/d)H_3^2] + C I_4' \tau_B [1 - A_2 \tau_B M_4^8 - B_2 \tau_B M_4^6] \quad (19)$$

where $H_n^m = J_m/J_n$, $M_n^m = I_m'/I_n'$,

$$I_r' = \int_{\theta^*/T}^{\theta/T} x^r e^x (e^x - 1)^{-2} dx, \quad r = 1, 2, 3 \ldots$$

Due to the low values of T and the high value of θ, the upper limit of integral I_r' can be taken as ∞; this integral can also be evaluated similarly to J_N and one gets

$$I_n' = \frac{1}{\mathrm{N!}} e^{-\theta/T} \sum_{n=0}^{N} \frac{1}{\mathrm{n!}} \left(\frac{\theta^*}{T}\right)^n .$$

For lower values of T for which $(\theta^*/T) > 10$, the contribution due to the second bracket in Eq. (19) is much smaller compared to that due to the first bracket. Thus for such values of T, the contribution due to the second bracket can be neglected and the expression for W_e can be approximated as

$$W_e = 2\pi^2 V K_B^{-4} \hbar^3 D T^{-2} J_3^{-1} [1 + (H_3^2 - H_3^4)(\tau_B^{-1}/D) T^{-1} + (H_3^6 - H_4^8 H_4^3)(K_B/\hbar)^4 (A/D) T^3 + (H_3^4 - H_4^6 H_4^5)(K_B/\hbar)^2 (B/D) T^4] . \quad (20)$$

For large values of the carrier concentration, the characteristic temperature θ^* of the doped sample is large enough, and at low temperatures, i.e. for $(\theta^*/T) > 15$, the expression for W_e in Eq. (20) can be further simplified as

$$W_e = 2.73 (V/K_B)(K_B/\hbar)^{-3} D T^{-2} [1 + 0.18(\tau_B^{-1}/D) T^{-1} - 332.8 (K_B/\hbar)^4 (A/D) T^3 - 4.26 (K_B/\hbar)^2 (B/D) T^4] . \quad (21$$

Results and discussion

Using the values of the constants reported in Table 1, which are taken from the report of Boghossian and Dubey [11], the extra lattice thermal resistivity W_e due to the presence of electrons of the five samples of phosphorus-doped Ge having different carrier concentrations in the range $1.2 \times 10^{23} - 1.1 \times 10^{24}$ m^{-3} has been calculated in the entire temperature range 1 – 5 K with the help of the numerical analysis of Eq. (5) and the results obtained are illustrated in Fig. 1. To compare W_e with the total lattice thermal resistivity of the corresponding undoped sample, W_{undoped} has also been calculated using Eq. (1), and the results obtained are reported in Fig. 1. The percentage contribution of W_e towards the total lattice ther-

Table 1

Values of constants and parameters used in the analysis of the extra lattice thermal resistivity W_e due to the presence of electrons of the five samples of P-doped Ge having different carrier concentrations in the entire temperature range 1—5 K

	Sample A		Sample B		Sample C		Sample D		Sample E	
n, m^{-3}	1.2×10^{23}		1.7×10^{23}		2.35×10^{23}		5.6×10^{23}		1.1×10^{24}	
E_D, J	2.40×10^{-19}		5.09×10^{-19}		6.30×10^{-19}		8.07×10^{-19}		8.13×10^{-19}	
θ^*, K	24		26.8		28		29.2		30	
T, K	η^*	m^*	η^*	m^*	η^*	m^*	η^*	m^*	η^*	m^*
1	25.63	0.24	25.97	0.24	29.30	0.25	30.96	0.27	31.60	0.28
2	21.43	0.32	22.35	0.35	24.20	0.37	25.70	0.39	26.35	0.40
3	19.36	0.38	19.73	0.40	21.15	0.42	22.28	0.44	23.07	0.45
4	18.35	0.40	18.76	0.42	19.50	0.44	20.25	0.45	20.80	0.47
5	17.92	0.41	18.36	0.43	18.90	0.45	19.45	0.47	20.00	0.48

$V = 3.9\times10^3$ m/sec,
$V_L = 4.92\times10^3$ m/sec,
$\theta = 376$ K,
$\tau_B^{-1} = 7.8\times10^5$ sec^{-1},
$A = 2.4\times10^{-44}$ sec^3.

Table 2

Percentage contribution % W_e of the extra lattice thermal resistivity due to the presence of electrons towards the total lattice thermal resistivity of P-doped Ge samples having different carrier concentrations in the temperature range 1—5 K

	Sample A $n = 1.2\times10^{23}$ m^{-3}	Sample B $n = 1.7\times10^{23}$ m^{-3}	Sample C $n = 2.35\times10^{23}$ m^{-3}	Sample D $n = 5.6\times10^{23}$ m^{-3}	Sample E $n = 1.1\times10^{24}$ m^{-3}
T, K	% W_e	% W_e	% W_e	% W_e	% W_e
1	86.99	95.16	97.18	98.49	98.72
2	88.65	94.81	96.76	97.97	98.30
3	88.87	93.14	95.27	96.70	97.21
4	87.95	92.12	93.97	95.10	95.98
5	85.75	91.28	93.06	94.52	95.12

mal resistivity of the above five samples of P-doped Ge has also been analysed and the variation of % W_e with the temperature T is reported in Table 2. To analyse the effects of the reduced Fermi energy η^*, the density of states effective mass m^*, the deformation potential constant E_D and the carrier concentration n on the extra lattice thermal resistivity W_e due to the presence of electrons, the variations of W_e with η^*, m^*, E_D and n are reported for the first time in Figs 2, 3, 4 and 5, respectively. The percentage contribution % W_e for the different values of η^* and

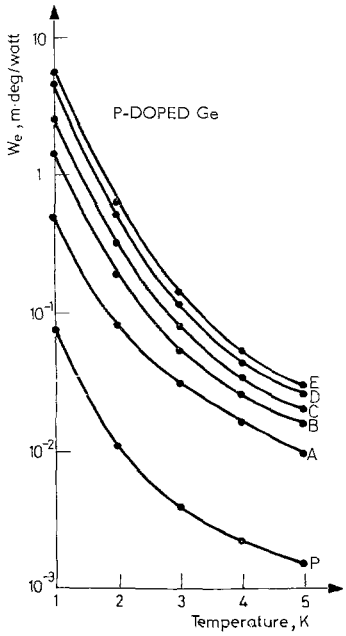

Fig. 1. The extra lattice thermal resistivity W_e due to the presence of electrons of five samples of P-doped Ge in the temperature range 1—5 K. Curves A, B, C, D and E correspond to samples having carrier concentrations 1.2×10^{23}, 1.7×10^{23}, 2.35×10^{23}, 5.6×10^{23} and $1.1\times$ $\times10^{24}$ m^{-3}, respectively. Curve P corresponds to the undoped sample. The solid line gives the calculated values, while circles are based on the experimental data

m^* has also been studied and the results obtained are reported in Tables 3 and 4 respectively. The variation of $\%\ W_e$ with the parameters E_D and n at constant temperature is given in Table 5. To examine the accuracy of the values of the constants used in the present analysis, the total lattice thermal conductivity of the sample of P-doped Ge having the carrier concentration $n = 1.2\times10^{23}$ m^{-3} has been calcu-

Table 3

Variation of $\%\ W_e$ with the reduced Fermi energy η^* for P-doped Ge

Sample A $n = 1.2\times10^{23}$ m^{-3}		Sample B $n = 1.7\times10^{23}$ m^{-3}		Sample C $n = 2.35\times10^{23}$ m^{-3}		Sample D $n = 5.6\times10^{23}$ m^{-3}		Sample E $n = 1.1\times10^{24}$ m^{-3}	
η^*	$\%\ W_e$	η^*	$\%\ W_e$	η^*	$\%\ W_e$	η^*	$\%\ W_e$	η^*	$\%\ W_e$
25.63	86.99	25.97	95.16	26.30	97.18	30.96	98.49	31.60	98.72
21.43	88.65	22.35	94.18	24.20	96.76	25.7	97.97	26.35	98.30
19.36	88.87	19.73	93.14	21.15	95.27	22.28	96.70	23.07	97.21
18.35	87.95	18.76	92.12	19.50	93.97	20.25	95.10	20.80	95.98
17.92	87.75	18.36	91.28	18.90	93.06	19.45	94.52	20.0	95.12

Table 4

Variation of $\% W_e$ with the density of the states effective mass m^* for P-doped Ge

Sample A $n = 1.2\times10^{23}$ m^{-3}		Sample B $n = 1.7\times10^{23}$ m^{-3}		Sample C $n = 2.35\times10^{23}$ m^{-3}		Sample D $n = 5.6\times10^{23}$ m^{-3}		Sample E $n = 1.1\times10^{24}$ m^{-3}	
m^*	$\% W_e$	m^*	$\% W_e$	m^*	$\% W_e$	m^*	$\% W_e$	m^*	$\% W_e$
0.24	86.99	0.24	95.16	0.25	97.18	0.27	98.49	0.28	98.72
0.32	88.65	0.35	94.18	0.37	96.76	0.39	97.97	0.40	98.30
0.38	88.87	0.40	93.14	0.42	95.27	0.44	96.70	0.45	97.21
0.40	87.95	0.42	92.12	0.44	93.97	0.45	95.10	0.47	95.98
0.41	87.75	0.43	91.28	0.45	93.06	0.47	94.52	0.48	95.12

Table 5

Variation of $\% W_e$ with the deformation potential constant E_D and the carrier concentration n at constant temperature for P-doped Ge

E_D, 10^{-19} J	$T = 1$ K	$T = 2$ K	$T = 3$ K	$T = 4$ K	$T = 5$ K	n (10^{23} m^{-3})
	$\% W_e$	$\% W_e$	$\% W_e$	$\% W_e$	$\% W_e$	
2.40	86.99	88.65	88.87	87.95	85.75	1.2
5.09	95.16	94.81	93.14	92.12	91.28	1.7
6.30	97.18	96.76	95.27	93.97	93.06	2.35
8.07	98.49	97.97	96.70	95.10	94.52	5.6
8.13	98.72	98.30	97.21	95.98	95.12	11

lated for comparison with the experimental data, and the results obtained are illustrated in Fig. 6; this shows a very good agreement between the calculated and the experimental values of the lattice thermal conductivity in the entire temperature range of study. To establish the relative importance of the scattering relaxation rates used in the present analysis, the variation of these scattering relaxation rates with the dimensional parameter x has been studied and the results obtained are reported in Fig. 7 for the sample having the carrier concentration $n = 1.2\times10^{23}$ m^{-3}.

(A) *Variation of the extra lattice thermal resistivity W_e due to electrons with temperature.*

The variation with temperature of the extra lattice thermal resistivity W_e due to the presence of electrons of the five samples of P-doped Ge having different carrier concentrations in the range $1.2\times10^{23} - 1.1\times10^{24}$ m^{-3} can be studied with the help of Fig. 1. In this Figure the experimental values are shown by circles, which mean the values obtained by subtracting the experimentally determined lattice thermal resistivity of the undoped sample from that of the corresponding doped

sample [30]. From this Figure, it is very clear that at low temperatures, W_e decreases with an increase of temperature T for each value of the carrier concentration. At the same time, it csan be seen that the variation of W_e with temperature at lower temperatures is faster than that at higher temperatures. From this Figure, one can also see that the nature of the W_e vs. T curve is fairly similar for each sample, and that the extra lattice thermal resistivity W_e of a doped sample due to the presence of electrons is much larger than the total lattice thermal resistivity of the corresponding undoped sample, which suggests that at low temperatures the electron-phonon scattering mechanism provides a very large contribution towards the total lattice thermal resistivity of a doped sample. As a result, it can be said that at low temperatures the reduction in lattice thermal conductivity of a doped sample is mainly due to the presence of electrons.

The percentage contribution $\%\ W_e$ due to the presence of electrons towards the total lattice thermal resistivity of P-doped Ge can be studied with the help of Table 2, which shows that, except for the sample having the carrier concentration $1.2\times10^{23}\ m^{-3}$, $\%\ W_e$ decreases with the increase of temperature. For the sample having $n = 1.2\times10^{23}\ m^{-3}$, $\%\ W_e$ shows an increasing nature with T below 3 K, but a decreasing nature with an increase in T above 4 K. These tendencies of $\%\ W_e$ can be analysed by considering the roles of τ_B^{-1} and τ_{ep}^{-1}.

(B) *Variation of W_e with the reduced Fermi energy η^**

The effect of the reduced Fermi energy η^* on the extra lattice thermal resistivity W_e of a doped sample due to the presence of electrons can be studied by analysing Fig. 2, which shows the variation of W_e with η^* for four samples of P-doped Ge having different carrier concentrations. The variation of W_e with η^* for the sample having $n = 5.6\times10^{23}\ m^{-3}$ could not be reported in this Figure for the sake of clarity. From this Figure, it is clear that W_e increases with increasing η^* for each sample, i.e. for each value of the carrier concentration. The increasing nature of W_e can be understood as follows. From the earlier report of Boghossian and Dubey as well as with the help of Table 1, it is clear that η^* shows a decreasing nature with increasing temperature and with the help of Fig. 1 as well as Eqs (18) and (21) it is very clear that W_e also shows a decreasing nature with T. As a result of these two variations, W_e should increase with increasing η^*, similarly to the results reported in Fig. 2. The variation of $\%\ W_e$ with η^* can be studied with the help of self-explanatory Table 3, which shows that, except for the sample having $n = 1.2\times10^{23}\ m^{-3}$, $\%\ W_e$ increases with increasing η^*, which is just the opposite to the variation of $\%\ W_e$ with T.

(C) *Variation of W_e with the density of the states effective mass m^**

Figure 3 can be used to study the effect of the density of the states effective mass m^* on the extra lattice thermal resistivity W_e of a doped semiconductor due to the presence of electrons. This Figure shows the variation of W_e with m^* for the five samples of P-doped Ge having different carrier concentrations in the range $1.2\times10^{24} - 1.1\times10^{23}\ m^{-3}$. From analysis of this Figure, one can say that the

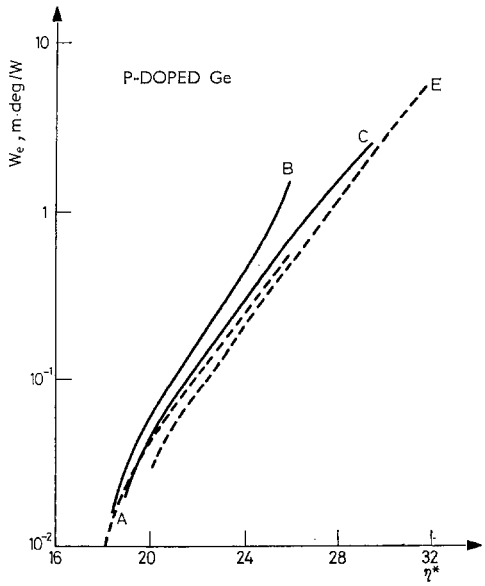

Fig. 2. Variation of the extra lattice thermal resistivity W_e due to the presence of electrons with the reduced Fermi energy η^* for four samples of P-doped Ge. Curves A, B, C and E correspond to samples having $n = 1.2\times10^{23}$, 1.7×10^{23}, 2.35×10^{23} and 1.1×10^{24} m^{-3}, respectively

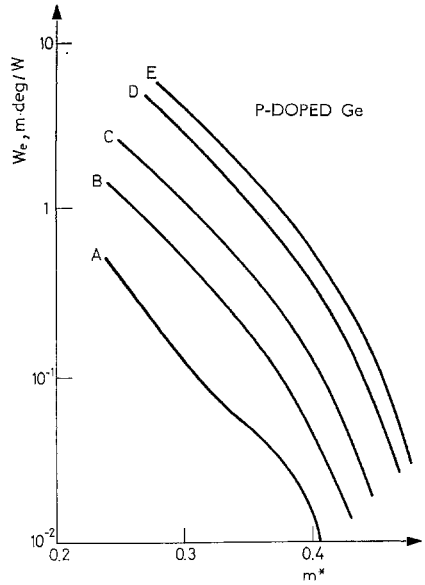

Fig. 3. Variation of the extra lattice thermal resistivity W_e due to the presence of electrons with the density of the states effective mass m^* for five samples of P-doped Ge. Curves A, B, C, D and E correspond to samples having $n = 1.2\times10^{23}$, 1.7×10^{23}, 2.35×10^{23}, 5.6×10^{23} and 1.1×10^{24} m^{-3}, respectively

lattice thermal resistivity W_e of a doped sample due to electrons decreases with an increase of m^*. At the same time, it can also be suggested that the variation of W_e with m^* is nearly the same for each sample. The decreasing nature of W_e with m^* can be explained with the help of the variation of m^* with temperature.

The density of states effective mass m^* of the electron is an important parameter in the estimation of the electron-phonon scattering relaxation rate τ_{ep}^{-1} and it depends on the carrier concentration n, as well as on the temperature, as

$$m^* = \left(\frac{n}{4\pi}\right)^{2/3} \frac{\hbar^2}{2m_0 K_B T} [F_{1/2}(\eta^*)]^{-2/3} \tag{22}$$

where m_0 is the rest mass of the electron and $F_{1/2}(\eta^*)$ is the Fermi integral. The temperature-dependence of m^* was studied by the author and his co-workers and it was found that m^* increases with increasing temperature, which can be confirmed using Table 1. At the same time, analysis of Fig. 1 suggests the decreasing nature of W_e with increasing T. As a result of these two variations, one can conclude that W_e must decrease with increase of the density of states effective mass m^* of electrons.

The effect of m^* on % W_e can be studied by analysing the results reported in Table 4, which shows that % W_e decreases with m^* for each sample except that having the carrier concentration $n = 1.2 \times 10^{23}$ m^{-3}, for which % W_e increases with temperature below 3 K, while it shows a decreasing nature with T above 3 K. It is useful to state that the variation of % W_e with m^* is approximately similar to the variation of % W_e with T.

(D) *Variation of W_e with E_D*

The deformation potential constant E_D, which is one of the responsible factors in the assignment of the electron-phonon scattering relaxation rate, depends mainly on the carrier concentration and the structure of the crystal, and it is interesting to study the effect of E_D on the extra lattice thermal resistivity W_e of a doped sample due to the presence of electrons. Figure 4 shows the variation of W_e with E_D at constant temperature for P-doped Ge. From this Figure, it can be seen that W_e increases with increase of the deformation potential constant E_D at each temperature. At the same time, it can also be seen that the nature of the W_e vs. E_D curve is almost the same at each temperature. The increasing nature of W_e with E_D can be explained with the help of Eqs (2), (18) and (21). The analytical expressions reported in Eqs (18) and (19) show that at constant temperature, $W_e \propto D$ for both of the cases $\tau_B^{-1} > \tau_{ep}^{-1}$ and $\tau_{ep}^{-1} > \tau_B^{-1}$. At the same time, from Eq. (2) it can be seen that the electron-phonon scattering constant D is directly proportional to E_D^2. As a result, one can say that $W_e \propto E_D^2$.

The effect of E_D on the percentage contribution % W_e towards the total lattice thermal resistivity of a doped sample can be studied with the help of the results reported in Table 5. From this Table, it is very clear that at low temperatures % W_e increases with increasing E_D for each value of T.

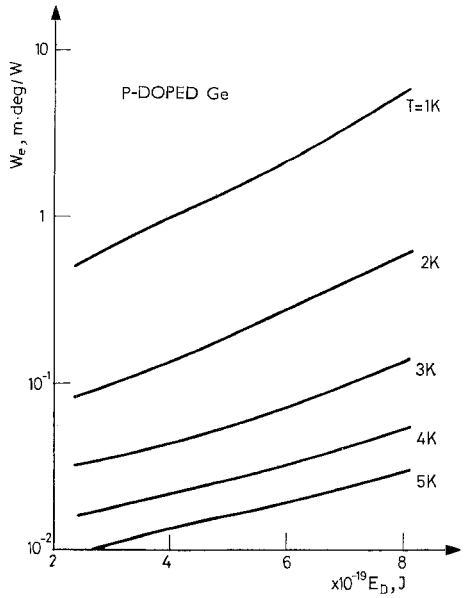

Fig. 4. Variation of the extra lattice thermal resistivity W_e due to the presence of electrons with the deformation potential constant E_D at constant temperature for P-doped Ge

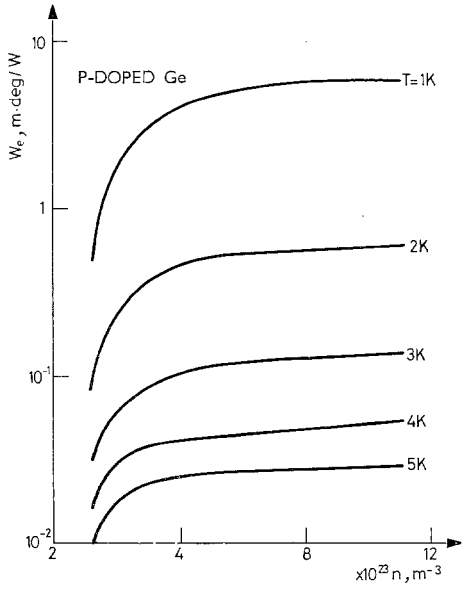

Fig. 5. Variation of the extra lattice thermal resistivity W_e due to the presence of electrons with the carrier concentration n at constant temperature for P-doped Ge

(E) *Variation of W_e with the carrier concentration n*

The expression for the electron-phonon scattering relaxation rate τ_{ep}^{-1} in Eq. (2) does not contain the carrier concentration term n. However, the parameters η^*, m^* and E_D which are responsible for τ_{ep}^{-1} are mainly governed by the carrier concentration n. At the same time, the characteristic temperature θ^*, which differentiates peripheral and non-peripheral phonons, plays a very important role in the estimation of W_e and depends on the carrier concentration n. Therefore, it is needed to study the effect of the carrier concentration on the extra lattice thermal resistivity W_e due to the presence of electrons of a doped sample.

From Fig. 5, which shows the variation of W_e of P-doped Ge with n, it can be seen that W_e shows an increasing nature with an increase of n at constant temperature. The nature of the W_e vs. n plot is nearly the same for each value of T. From this Figure, it is also very clear that the variation in W_e is much faster at lower values of n than for larger values of n. An increase in the carrier concentration n means an increase in the number of scatterers in the crystal. As a result the number of interactions increases, which causes an enhancement in the lattice resistivity.

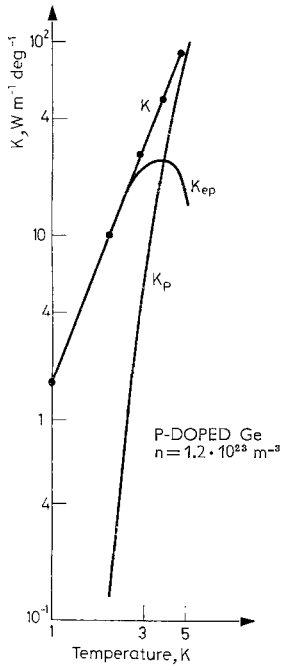

Fig. 6. Total lattice thermal conductivity of the P-doped Ge sample having the carrier concentration 1.2×10^{23} m^{-3} in the entire temperature range 1– 5 K. K is the total lattice thermal conductivity, K_{ep} is the lattice thermal conductivity due to non-peripheral phonons and K_{ph} is the same due to peripheral phonons. The solid line gives the calculated values, while the circles are the experimental points

Table 5 clearly demonstrates that at low temperatures % W_e increases with increase of the carrier concentration n for each value of T, which is similar to the variation of % W_e with E_D.

(F) *Reliability of constants and scattering relaxation rates*

The reliability of the values of the constants used in the present analysis can be tested with the help of Fig. 6, which shows the total lattice thermal conductivity of the P-doped Ge sample having the carrier concentration $n = 1.2 \times 10^{23}$ m^{-3} in the entire temperature range 1 – 5 K. From this Figure, it is very clear that the values of the constants used in the present analysis give excellent agreement between the calculated and experimental values of the lattice thermal conductivity in the entire temperature range 1 – 5 K, which predicts that these values are correct. Using these values, the variation of τ_B^{-1}, τ_{pi}^{-1} and τ_{ep}^{-1} with the dimensionless parameter x is illustrated at constant temperature for the P-doped Ge sample having $n = 1.2 \times 10^{23}$ m^{-3} in Fig. 7. The variation of τ_{ph}^{-1} with x could not be included in

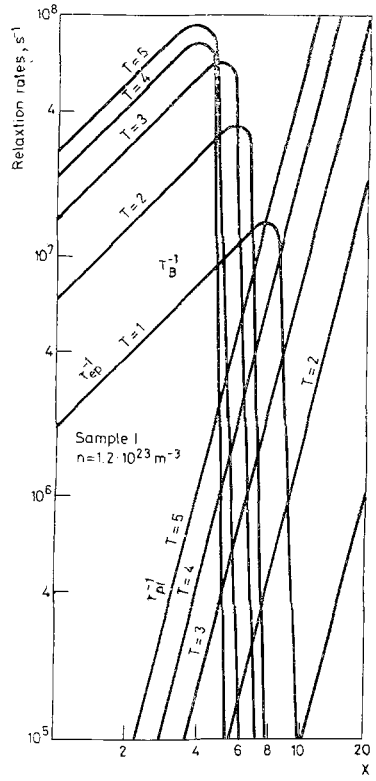

Fig. 7. Variation of the scattering relaxation rates τ_B^{-1}, τ_{pi}^{-1} and τ_{ep}^{-1} with the dimensionless parameter x

this Figure due to its very small contribution compared to the other scattering relaxation rates. The dominating nature of τ_{ep}^{-1} over the other scattering relaxation rates can be seen with the help of Fig. 7, and this is very important in establishing the analytical expression for W_e.

Conclusions

The extra lattice thermal resistivity W_e due to the presence of electrons of a doped sample has been studied at low temperatures for the first time by calculating W_e for five samples of P-doped Ge having different carrier concentrations in the range $1.2 \times 10^{23} - 1.1 \times 10^{24}$ m^{-3} in the temperature range 1 – 5 K, and the results obtained are discussed above. With the help of these results, one can conclude the following:

1. The extra lattice thermal resistivity W_e due to the presence of electrons of a doped sample decreases with increasing temperature and % W_e also shows a decreasing nature with T.
2. W_e and % W_e show increasing natures with increase of the reduced Fermi energy η^* at a constant carrier concentration n.
3. Both W_e and % W_e exhibit decreasing tendencies with an increase in the density of the states effective mass m^* for a constant carrier concentration n.
4. At a constant temperature in the range 1 – 5 K, W_e and % W_e show increasing natures with increase of the deformation potential constant E_D.
5. At constant temperature, W_e and % W_e increase with the carrier concentration n.
6. The analytical expression has been obtained for calculation of an approximate value of W_e, and it is found that $W_e \propto T^2$ for both the cases $\tau_B^{-1} > \tau_{ep}^{-1}$ and $\tau_B^{-1} < \tau_{ep}^{-1}$.

*

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References

1. J. M. ZIMAN, Phil. Mag., 1 (1956) 191.
2. R. W. KEYES, Phys. Rev., 122 (1961) 1171.
3. A. GRIFFEN and P. CARRUTHERS, Phys. Rev., 131 (1963) 1976.
4. P. C. KWOK, Phys. Rev., 149 (1966) 666.
5. N. K. S. GAUR and G. S. VERMA, Phys. Rev., 159 (1967) 610.
6. M. P. SINGH and G. S. VERMA, Physica, 62 (1972) 627.
7. A. KUMAR and M. M. JOSHI, Phys. Rev., B4 (1971) 4643.
8. J. F. GOLF and N. PEARLMAN, Phys. Rev., 140 (1965) 2151.
9. R. S. BLEWER, N. H. ZEBOUNI and C. G. CRENEIR, Phys. Rev., 174 (1968) 700.
10. K. S. DUBEY and G. S. VERMA, Phys. Rev., B4 (1971) 2071.

11. H. H. BOGHOSSIAN and K. S. DUBEY, Phys. Stat. Solidi, b88 (1978) 417.
12. H. H. BOGHOSSIAN and K. S. DUBEY, Sol. Stat. Comms., 27 (1978) 1065.
13. M. G. HOLLAND, Phys. Rev., 132 (1963) 2461.
14. K. S. DUBEY and G. S. VERMA, Phys. Rev., B4 (1971) 4491.
15. K. S. DUBEY and R. H. MISHO, J. Thermal Anal., 15 (1978) 223.
16. K. S. DUBEY, J. Thermal Anal., 18 (1980) 35.
17. K. S. DUBEY, J. Thermal Anal., 19 (1980) 263.
18. J. CALLAWAY, Phys. Rev., 113 (1959) 1046.
19. H. B. G. CASIMIR, Physica, 5 (1938) 495.
20. P. G. KLEMENS, Sol. Stat. Phys., 7 (1958) 1.
21. C. HERRING, Phys. Rev., 95 (1954) 954.
22. K. S. DUBEY, J. Phys. 47 (1976) 263.
23. K. S. DUBEY, Sol. Stat. Comms., 23 (1977) 963.
24. K. S. DUBEY, Phys. Stat. Solidi, B79 (1977) 119.
25. K. S. DUBEY, Phys. Stat. Solidi, B81 (1977) 83.
26. H. H. BOGHOSSIAN and K. S. DUBEY, Phys. Stat. Solidi, B96 (1979) K107.
27. K. S. DUBEY, Ind. J. Pure. Appl. Phys., 11 (1973) 182.
28. K. S. DUBEY, Phys. Stat. Solidi, B82 (1977) K63.
29. K. S. DUBEY, Ind. J. Pure Appl. Phys., 11 (1973) 179.
30. M. P. MATHUR and N. PEARLMAN, Phys. Rev., 180 (1969) 833.

RÉSUMÉ — La variation en fonction de la température de la résistance thermique extra du réseau d'un échantillon dopé, due aux électrons, a été étudiée pour la première fois à basse température, en analysant la résistance thermique extra du réseau due aux électrons de cinq échantillons de Ge dopés en phosphore à diverses concentrations comprises entre 1.2×10^{23} et 1.1×10^{24} m^{-3} dans l'intervalle de températures de 1 à 5 K. Dans le cadre de cette étude on a analysé, également pour la première fois, la variation de la résistance extra du réseau d'un échantillon dopé due aux électrons, avec les paramètres η^* (énergie de Fermi réduite), m^* (densité d'état de la masse effective), E_D (constante du potentiel de déformation) et n (concentration du support), qui sont responsables de la vitesse de relaxation de la diffusion phonons — électrons. On décrit une expression analytique pour le calcul approché de la résistance thermique extra du réseau, à basse température, d'un échantillon dopé, due aux électrons.

ZUSAMMENFASSUNG — Die Temperaturabhängigkeit des auf die Gegenwart von Elektronen zurückzuführenden zusätzlichen thermischen Widerstandes des Gitters einer dotierten Probe wurde bei niedrigen Temperaturen durch die Analyse des durch Elektronen hervorgerufenen zusätzlichen thermischen Widerstandes bei fünf mit Phosphor dotierten Ge Proben mit unterschiedlicher Trägerkonzentration von $1.2\times 10^{23} - 1.1\times 10^{24}$ m^{-3} im Temperaturbereich von 1—5 K, erstmalig untersucht. Die Änderung des durch Elektronen hervorgerufenen zusätzlichen thermischen Widerstandes des Gitters einer dotierten Probe mit den Parametern η^* (normierte Fermienergie), m^* (effektive Masse der Zustandsdichte), E_D (Konstante des Deformationspotentials) und n (Trägerkonzentration), welche für den Anteil der Relaxation der Elektronen-Phononen-Streuung verantwortlich sind, wurde ebenfalls in der vorliegenden Untersuchung erstmals analysiert. Es wurde auch zwischen den nichtperipherischen und den peripherischen Phononen unterschieden. Ein analytischer Ausdruck zur Berechnung des Näherungswertes des zusätzlichen thermischen Widerstandes des Gitters durch die Gegenwart von Elektronen einer dotierten Probe bei niedrigen Temperaturen wird angegeben.

Резюме — В области низких температур изучена температурная зависимость сверхрешеточного удельного сопротивления, обусловленного наличием электронов, легированных образцов. Впервые исследование проводилось на пяти образцах германия, легированного фосфором, с концентрацией носителя в области $1.2 \cdot 10^{23}$—$1.10 \cdot 10^{24}$ м$^{-3}$ и при температурах 1—5 К. В работе впервые изучено изменение сверхрешеточного удельного сопротивления, обусловленного электронным вкладом легированных образцов с параметрами: η^* — уменьшение энергии Ферми, $м^*$ — плотность состояний эффективной массы, $Ед$ — константа деформационного потенциала и n — концентрация носителя, которые ответственны за скорость релаксации электрон-фононного рассеяния. В представленном анализе сделано различие между ближними и дальними фононами. Представлено аналитическое выражение для вычисления приближенного значения сверхрешеточного теплового удельного сопротивления, обусловленного наличием электронов в легированных образцах.